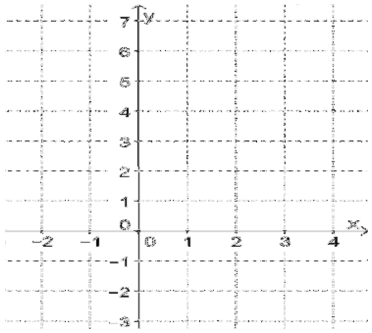
4.1 Relations Vs. Functions

Warm up:

Make a table of values, then graph the relation
y = 2x + 1

X	Y
-2	
-1	
0	
1	
2	
3	



Function vs Relation

Definition:

A *relation* is a rule which takes an input and may/or may not produce **multiple** outputs.

A *function* is a special relation where every input has **at most** one output.

Notation:

Relation: y = 2x + 1

Read it "f of x": the brackets here DO NOT MEAN MULTIPLY!!

Function: f(x) = 2x + 1

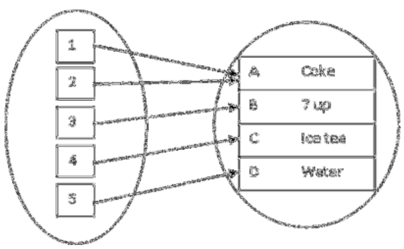
Ex 1: find f(7) = 2(7) + 1
= 15

find f(-5) = 2(-5) + 1
= -9

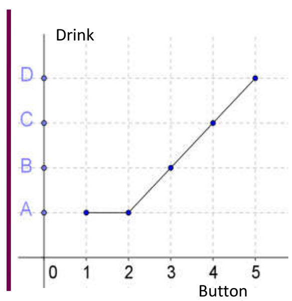
Consider 2 machines. **Machine A:**

Buttons
Push **IN** to order

Drink
Comes **OUT**



Vertical line test (VLT)

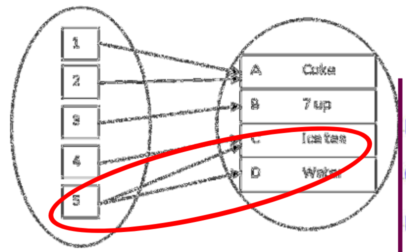


A FUNCTIONING MACHINE

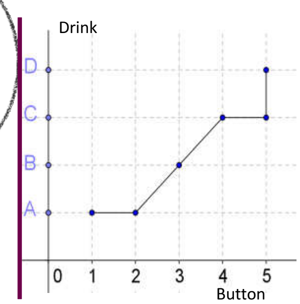
Machine B:

Buttons
Push **IN** to order

Drink
Comes **OUT**



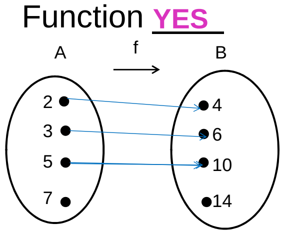
Vertical line test (VLT)



A Not FUNCTIONING MACHINE

(1) Mapping Diagram

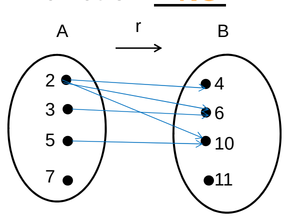
Function YES



Source
Input
Independent
X-values
Antecedent
Domain

Target
Output
Dependent
Y-values
Image
Range

Function NO



Each source value has only ONE target value

A source value has MORE THAN ONE target value

(2) Table of Values

Function YES

x	y
0	3
1	4
3	6
5	8

Function NO

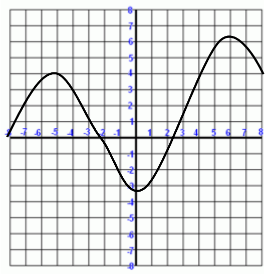
x	y
1	0
2	1
4	3
4	5

Each x value has only ONE y value.

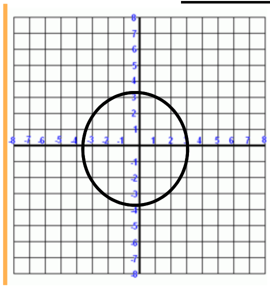
An x value has MORE THAN ONE y value

(3) Graph - Vertical Line Test (VLT)

Function YES



Function NO



A vertical line touches the curve at ONLY 1 spot at a time

A vertical line touches MORE THAN 1 spot at a time.

Ex 2: Evaluate the following functions

a) $f(x) = 3x + 5$ find $f(3)$

b) $g(x) = x^2 + 1$ find $g(6)$

c) $h(x) = (x-2)(x-5)$ find $h(6)$

Practice:
Pages 94,95 # 2-6

4.2 –A- Modes of Representation of a function

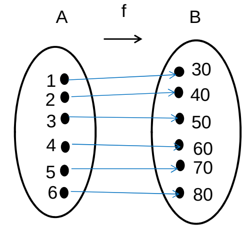
In a relation between 2 variables x and y, one usually depends on the other (the output depends on the input).

We say: y depends on x

- therefore y is the dependent variable,
- and x is the independent variable.

Do activities 1,2,3 on pages 96, 97

P. 96 Act. 1: A ferry ensures the transportation from a town to an island. The rates are the following: $20 per car and $10 per occupant in the car. No car is accepted without an occupant and there is a maximum of 6 occupants per car.



P.96 Act.2: Anna works as a dental hygienist in a clinic. Her hourly wage is $22. In this situation consider the following two variables: the number of hours worked in a week and her salary.

P. 97 Act 3: A water reservoir contains 1000 liters of water. A pump is activated to empty the reservoir at a rate of 50 liters per minute. Consider the function which associates the variable "elapsed time" with the variable "quantity of water left in the reservoir".

4.2-B- Modes of Representation of a function

There are different ways of representing a function:

1. Verbal/Written:

- That is a sentence/paragraph to describe the function in words.

Ex: A repairman charges $30 per hour plus $60 for his travel expenses.

2. Rule/Equation:

- That translates from English to Math, and expresses the dependent variable y in terms of the independent variable x.

Ex:

3. Table of Values:

- A way to organize data. It associates the x values with their y values.

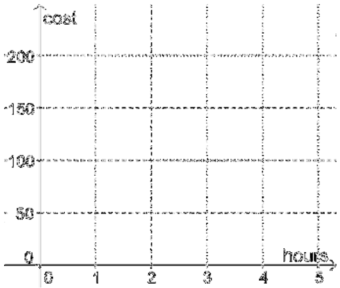
Ex:

Hours	0	1	2	3	4	5
Cost						

4. Cartesian graph or Mapping Diagram:

- A visual Representation

Ex:



Remarks:

1. The indep. Var. x goes on the Horizontal axis;
   the dep. Var. y goes on the vertical axis.
2. Choose an appropriate scale for the x and y-axis.
3. We can break the axis if the graph starts up too high.
4. Remember to label the axis, the scales, and put a title.

Practice:
Page 98 # 1

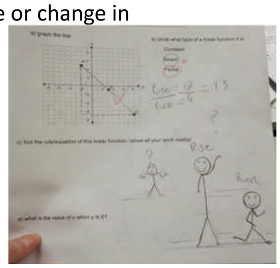
4.3 -A- Rate of change

- Rate of change (R.O.C) is also known as Rate of variation, Slope, and Rise over Run
- Given points A(x_1,y_1) & B(x_2,y_2) on a line, then the rate of change between points A & B is :

R.O.C. = $\frac{\Delta y}{\Delta x}$; Δ: difference or change in

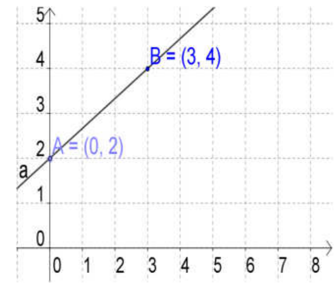
= $\frac{y_2 - y_1}{x_2 - x_1}$

or = $\frac{rise}{run}$ if you have a graph



Ex 1: Given points A(0,2) and B(3,4)

- R.O.C. = $\frac{\Delta y}{\Delta x}$

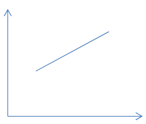


- Or Count $\frac{rise}{run}$ =

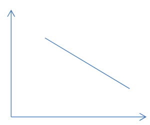
On any straight line all segments of the line will have the same slope

- 4 possible slopes are noted:

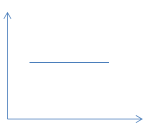
1. Positive slope :



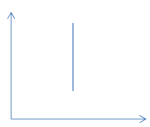
2. Negative slope:



3. Zero slope:

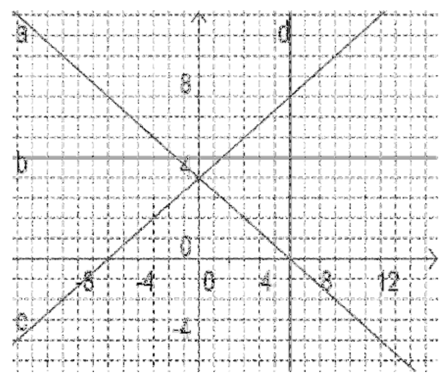


4. Undefined slope:



Ex 2: Find the slope of each line on the grid:

- Slope of line:
- a:
- b:
- c:
- d:



Ex 3: Find the slope of the following:

1) A(3,3) B(2,5)

$\frac{rise}{run} = \frac{y_2-y_1}{x_2-x_1}$

=

=

=

2) C(-1,5) D(4,8)

$\frac{rise}{run} = \frac{y_2-y_1}{x_2-x_1}$

Practice:
page 103 # 1-3

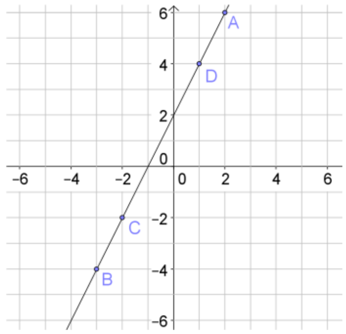
4.3 -B- Constant Rate of Change

Find the rate of changes:

a) Between A and D:

b) Between D and C:

c) Between C and A:



d) Between A and B:

Property of ROC of a straight line:

- On a straight line the ROC is constant (the same) no matter which 2 points we pick.
  - Where as on a curve the ROC changes (is not constant) along the curve.
- On a graph we can count the units:
  - ROC = $\frac{rise}{run}$
- Given 2 points A(x_1,y_1) and B(x_2,y_2):
  - ROC = $\frac{y_2 - y_1}{x_2 - x_1}$

Find the rate of Change for the Following

Ex 1: A tank initially contains 56 000 L of eggnog. The tank is leaking!
After 5 hrs the tank has 45 000 L .

Find the rate of Change for the Following

Ex 2: Jen earns $220 for 20 hours of work. For 10 hours of work she earns $120. What is her hourly rate?

x y

Hours	Pay
10	120
20	220

Find the value of c

Ex 3: The slope is 2
G(0,-5) and B(c,3)

Ex 4: The slope is 1/2
S(-1,2) and T(c,6)

Practice:
page 106 # 4-7

4.4 -A- Linear Functions

- Linear Functions have degree 1
- The rule: **y = ax + b**

  or **f(x) = ax + b**

Case 1: if b = 0 → Direct Variation Linear function

Case 2: if b ≠ 0 → Partial Variation Linear function

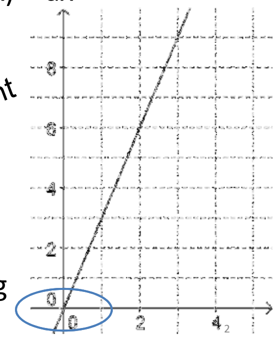
Case 1: if b=0 Direct Variation Linear function

Properties:

1. Every y-value is a direct multiple of the x-value
2. The rule: y = ax or f(x) = ax
3. Table of values:

x	y	1st Difference (Constant ROC)
0	0	
1	3	3
2	6	3
3	9	3

4. Graph: a Diagonal line passing through origin (0,0)



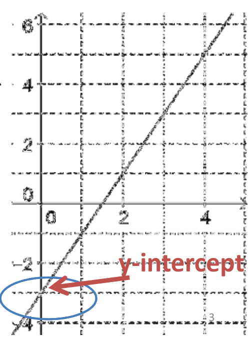
Case 2: if b≠0 Partial Variation Linear function

Properties:

1. y-values are not direct multiples of the x-values
2. The rule: y = ax + b or f(x) = ax + b
3. Table of values:

x	y	1st Difference (Constant ROC)
0	-3	
1	-1	2
2	1	2
3	3	2

4. Graph: a Diagonal line passing through y-axis at point (0,b)



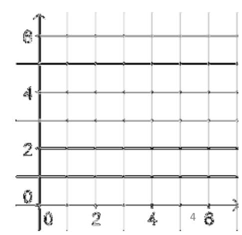
Constant Functions

Properties:

1. The ROC = 0 (∴ aka: zero variation function)
2. The rule: y = b or f(x) = b
3. Table of values:

x	y	1st Difference
0	5	
1	5	0
2	5	0

4. Graph: Horizontal line passing through the y-axis at b.



Ex 1: The movie ticket costs $9 for all ages.

If x is the age and y is the cost of ticket

- Table of values:
- ROC:
- The rule is :
- The graph:

Now try activity 2 Page 107

Determine the **degree** of each function.

f(x)= 2x+3 ___1___ f(x)= 5 ___0___

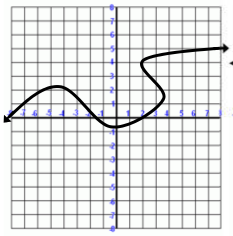
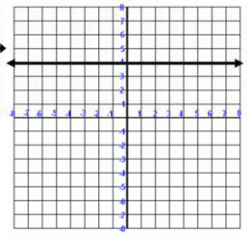
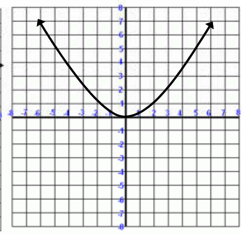
f(x)= $3x^2$ -2x + 1 ___2___ f(x)= $2x^3$ + 3x ___3___

f(x)= -4x+1 ___1___ f(x)= $\frac{1}{x}$ ___none___

The degree of a function determines the type of function

Degree	Type of function
0	Constant
1	Linear → Direct Linear function / Partial Linear function
2	Quadratic

Ex 1: is it Constant, Direct Linear, Partial Linear or Other

Ex 1: is it Constant, Direct Linear, Partial Linear or Other

x	y
1	3
2	5
3	7
4	9

x	y
10	20
9	18
8	16
7	14

x	y
0	5
20	25
40	45

Ex 1: is it Constant, Direct Linear, Partial Linear or Other

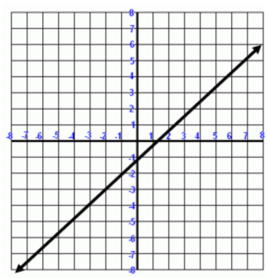
$y = 4x + 1$

$y = x^2$

$y = \sqrt{x}$

$f(x) = 3x$

$f(x) = 5$



Practice:
Day 1: Page 112 # 1-4

Tips for graphing a linear function:

1) Make a Table of Values (min 3 points)

2) Choose easy x-values like 0, 1 or 2.

If your slope is a fraction, pick multiples of the value for the **run (denominator)**.

- ie if slope = $\frac{3}{4}$ pick x =0, 4, 8.

1. Graph and label these equations

$y = 3x$ $\quad y = -3x + 2$ $\quad y = \frac{1}{3}x$ $\quad y = -\frac{1}{3}x - 3$

$y = 3x$

$y = \frac{1}{3}x$

$y = -3x + 2$

$y = -\frac{1}{3}x - 3$

1. Graph and label these equations

$y = 3x$ $\quad$ $y = -3x + 2$ $\quad$ $y = \frac{1}{3}x$ $\quad$ $y = -\frac{1}{3}x - 3$

$y = 3x$

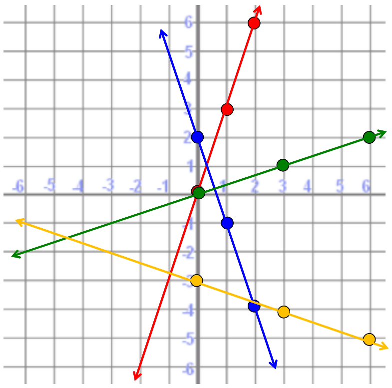
a = $\frac{3}{1}$ = $\frac{\text{rise}}{\text{run}}$

b = 0

$y = -3x + 2$

a = $\frac{-3}{1}$

b = 2



$y = \frac{1}{3}x$

a = $\frac{1}{3}$

b = 0

$y = -\frac{1}{3}x - 3$

a = $\frac{-1}{3}$

b = -3

Practice:

Day 2: Page 113 # 5-7

Day 3: Page 116 # 8-12

4.5 Finding the Rule of the Linear function

A straight line always follows the RULE

$$y = ax + b$$

Where:

- y → is the Dependent variable
- x → is the Independent variable
- a → is the R.O.C.(slope)
- b → is the initial value (y-intercept)

Steps to Finding the RULE given 2 points

Step 1: Find the slope using $a = \frac{y_2 - y_1}{x_2 - x_1}$

Step 2: Find the y-intercept (b) by plugging the (x_1, y_1) coordinates into $b = y_1 - ax_1$

Step 3: State the final equation.
$y = ax + b$

Ex1: Find the rule of the line going through (-6,5) & (-4, 6)

Step 1: Find a

Step 2: Find b using (-6,5)

Ex 2: Find the rule of the line going through (-2,6) & (1, 3)

A table of values shows the relationship between two variables, typically x and y.

time	height
0	1.5
1	3
2	4.5
3	6

We call

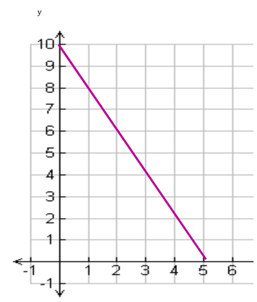
X the independent variable because we choose it.

y is the dependant variable because it depends on the chosen value of x.

TOV Case 1- Both a and b are clear

Run / Rise

x	y
0	10
1	8
2	6
3	4



a= ____ b= ____ y =__________

TOV Case 2- a is clear, must find b

Run / Rise

x	y
5	37
7	47
9	57
11	67

a= __ b= ___ y =________

TOV Case 3- a not clear and must find b

Run / Rise

x	y
32	59
14	23
5	5
73	141

$\frac{Rise}{Run}$ =

a= __ b= ___ y =________

Practice:
Page 120 # 1-7

4.6 Systems of 1st Degree Equations

- Finding the solution to a system of equations means find a common point (x,y), that fits into both equations at the same time.
- We can find the solution by making a table of values and finding when the values for y are the same.
- We can check the solution of a system by replacing it back into the original equations to see if it works.
- We can also graph the two lines on the same grid and see where the lines cross.

Ex 1: Is x = 2 and y = 4 a solution to the following systems?

1. $y = 2x$
   $y = x + 3$

2. $y = 6 - x$
   $y = x + 2$

Ex 2: Solve the system using a table of values

$y = 2x + 5$

$y = x + 8$

Choose values for x and calculate values for y . The solution exists when both values of y are the same.

x	$y_1 = 2x + 5$	$y_2 = x + 8$
0		
1		
2		
3		

Ex 3: Solve the system in ex 2, by graphing

$y = 2x + 5$

$y = x + 8$

x	$y_1 = 2x + 5$	$y_2 = x + 8$
0	5	8
1	7	9
2	9	10
3	11	11

4.6 Systems of 1st Degree Equations

- Finding the solution to a system of equations means find a common point (x,y), that fits into both equations at the same time.
- We can find the solution by making a table of values and finding when the values for y are the same.
- We can check the solution of a system by replacing it back into the original equations to see if it works.
- We can also graph the two lines on the same grid and see where the lines cross.

Ex 4: Solve using the comparison method

$y_1 = 4x + 6$

$y_2 = -2x$

Both equations must be isolated for the same variable

Compare both y's

Solve for x

Replace x and solve for y_1 then in y_2 to check

$\therefore$ *solution is*

Ex 5: Solve using the comparison method

$y = 3x - 2$
$y = 5x + 6$

∴ solution is (

Practice:
page 124 # 1,2
page 125 # 3-7

4.7 Parameters a and b in y = ax + b

- For every line y = ax + b

(the parameters are a and b, and they each affect the look of the line)

- We can observe the affects of changing the parameters by using technology:

TI-83 or GeoGebra or graphsketch.com

A	$y_1 = x$	$y_2 = 2x$	$y_3 = 0.5x$
type of function			
R.O.C. (a)			
Initial value (b)			
Description of line			
B	$y_4 = -x$	$Y_5 = -2x$	$y_6 = -0.5x$
type of function			
R.O.C. (a)			
Initial value (b)			
Description of line			

C	$y_1 = x$	$y_7 = x + 2$	$y_8 = x - 4$
type of function	**Direct**		
R.O.C. (a)	**1**		
Initial value (b)	**0**		
Description of line	**Centered in 1st and 3rd quadrants. Increasing line.**		
D	$y_9 = 3x + 2$	$y_{10} = 0.5x - 4$	$y_{11} = -2x + 6$
type of function			
R.O.C. (a)			
Initial value (b)			
Description of line			

Conclusions:

- For every line y = ax + b (the parameters are a and b affect the look of the line)
- a : affects the angle of inclination (steepness of line)
- b : affects the vertical translation of the line

4.7 Parameters a and b in y = ax + b

Observations on changing the parameters using: TI-83 or GeoGebra or graphsketch.com

A	$y_1 = x$	$y_2 = 2x$	$y_3 = 0.5x$
type of function	**Direct**	**Direct**	**Direct**
R.O.C. (a)	**1**	**2**	**0.5**
Initial value (b)	**0**	**0**	**0**
Description of line	**Centered in 1st and 3rd quadrants. Increasing line.**	**Steeper than y_1. Increases faster. Bigger angle of inclination.**	**Less steep than y_1. Increases slower. Smaller angle of inclination.**
B	$y_4 = -x$	$Y_5 = -2x$	$y_6 = -0.5x$
type of function	**Direct**	**Direct**	**Direct**
R.O.C. (a)	**- 1**	**- 2**	**- 0.5**
Initial value (b)	**0**	**0**	**0**
Description of line	**Centered in 2nd and 4th quadrants. Reflection of y = x. Decreasing line.**	**Steeper than y_1. decreases faster Bigger angle of inclination.**	**Less steep than y_1. decreases slower. Smaller angle of inclination.**
C	$y_1 = x$	$y_7 = x + 2$	$y_8 = x – 4$
type of function	**Direct**	**Partial**	**Partial**
R.O.C. (a)	**1**	**1**	**1**
Initial value (b)	**0**	**2**	**- 4**
Description of line	**Centered in 1st and 3rd quadrants. Increasing line.**	**Parallel to y_1 Translated (shifted) up 2 units**	**Parallel to y_1 Translated (shifted) down 4 units**
D	$y_9 = 3x + 2$	$y_{10} = 0.5x – 4$	$y_{11} = -2x + 6$
type of function	**Partial**	**Partial**	**Partial**
R.O.C. (a)	**3**	**0.5**	**- 2**
Initial value (b)	**2**	**- 4**	**6**
Description of line	**3 times steeper than y_1 Shifted up 2 units**	**Half as steeper as y_1 Shifted down 4 units**	**2 times steeper than y_1 and reflected Shifted up 6 units**

Conclusions: For every line y = ax + b (the parameters are a and b affect the look of the line)

a : affects the angle of inclination (steepness of line)

b : affects the vertical translation of the line

4.8 Rational function

Act. 1 Page 133: Savannah wants to repaint the offices at work. The job requires 40 hours of work for one employee. In this situation, consider the function f which associates the number x of employees hired for the job with the time y that it takes to complete the job.

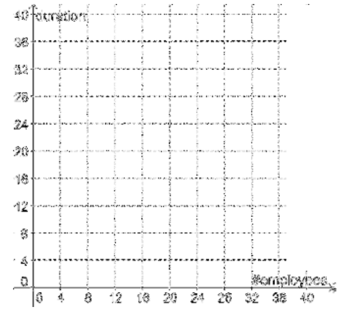
a

# emplo-yees x	Duration (in hours) y
1	
2	
4	
8	
10	
20	
40	

4.8 Rational function

Act. 1 Page 133: Savannah wants to repaint the offices at work. The job requires 40 hours of work for one employee. In this situation, consider the function f which associates the number x of employees hired for the job with the time y that it takes to complete the job.

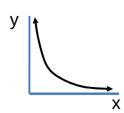
# emplo-yees x	Duration (in hours) y
1	40
2	20
4	10
8	5
10	4
20	2
40	1



In a Rational Function

- Variables x and y are inversely proportional

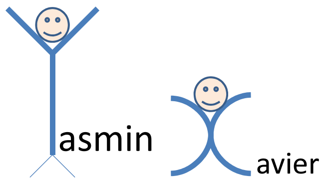
  That is as x increases y decreases.
- The rate of change is not constant.
- The product of each pair x and y is constant.
- The rule is: $y = \frac{k}{x}$ because xy= k
- The graph looks like



Practice:
Page 134 # 1-7

4.9 Inverse of a function

Imagine 2 friends:



Compare	If	Then
Age	$y = x$	
Height	$y = 2x$	
Marks	$y = x + 5$	
Stamp collection	$y = x - 10$	
Game score	$y = 3x - 2$	

Sometimes we need to inverse a function to express it in terms of the other variable.

Examples:

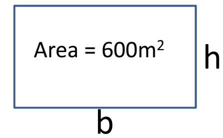
1) If $P = 4x$

Then x =

2) If $A = 600\ m^2$

Then h =

And b =

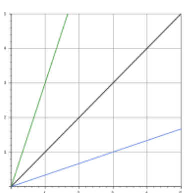


3) If $C = 10 + 2n$

Then n =

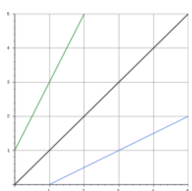
1. In a Direct Variation situation

If $y = ax$ then $x = \frac{y}{a}$



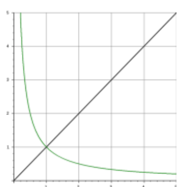
2. In a Partial Variation situation

If $y = ax + b$ then $x = \frac{y - b}{a}$



3. In a Rational Variation situation

If $y = \frac{c}{x}$ then $x = \frac{c}{y}$



To find the <u>inverse</u> of a function: Swap the x's and y's of each co-ordinate.

Ex. 1:

Function A

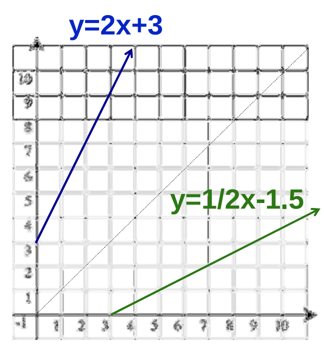
x	y
0	3
1	5
2	7
3	9
4	11

$f(x)$

Inverse of A

x	y
3	0
5	1
7	2
9	3
11	4

$f^{-1}(x)$



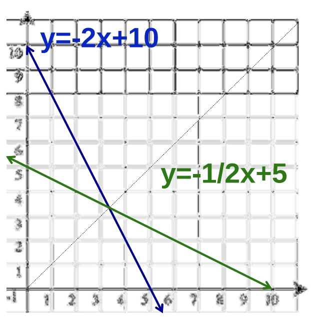
Ex 2: Graph $y = -2x + 10$ and its inverse

Function A

x	y
0	
1	
2	
3	
4	
5	

Inverse of A

x	y



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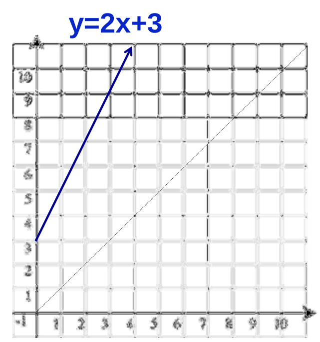

Ex 2: Graph $y = -2x + 10$ and its inverse

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x	y
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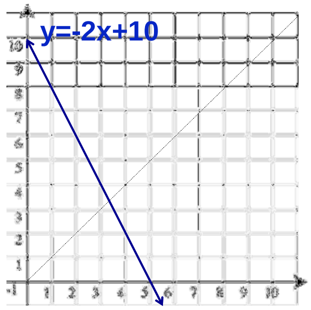
Inverse of A

x	y



Practice:
Page 140 # 1, 3, 5